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Analysis of two-layered micro-channel heat sink concept in electronic cooling

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Abstract

In this work, a new concept for a two-layered micro-channel heat sink with counter current flow arrangement for cooling of the electronic components is proposed. The thermal performance and the temperature distribution for these types of micro channels were analyzed and a procedure for optimizing the geometrical design parameters is presented. While the power supply system of the two-layered design is not significantly more complicated than the one-layered design, the streamwise temperature rise on the base surface was found to be substantially reduced compared to that of the one-layered heat sink. At the same time, the pressure drop required for the two-layered heat sink was found to be substantially smaller than that of the one-layered heat sink. The results demonstrate that the two-layered micro-channel heat sink design is a substantial improvement over the conventional one-layered micro-channel heat sink. © 1999 Elsevier Science Ltd. All rights reserved.

Nomenclature

 $\begin{array}{lll} c_{\rm p} & {\rm thermal\ capacity}\ [{\rm J\ kg^{-1}\ K^{-1}}] \\ H_{\rm ch} & {\rm channel\ height}\ [\mu{\rm m}] \\ H_{\rm ba} & {\rm base\ thickness}\ [\mu{\rm m}] \\ k & {\rm thermal\ conductivity}\ [{\rm W\ m^{-1}}] \\ L & {\rm channel\ length}\ [\mu{\rm m}] \\ m & {\rm mass\ flow\ rate}\ [{\rm kg\ s^{-1}}] \\ P & {\rm pressure\ [N\ m^{-2}]} \\ \Delta P & {\rm pressure\ Grop\ across\ a\ channel\ [N\ m^{-2}]} \\ q & {\rm applied\ heat\ flux\ [W\ cm^{-2}]} \\ Re & {\rm Reynolds\ number} \\ R_{\rm th}^{\rm n} & {\rm normal\ thermal\ resistance}\ [{\rm K\ W^{-1}}] \\ R_{\rm th}^{\rm r} & {\rm thermal\ resistance}\ [{\rm K\ W^{-1}}] \\ W_{\rm ch} & {\rm channel\ half\ width}\ [\mu{\rm m}] \\ W_{\rm fin} & {\rm fin\ half\ width}\ [\mu{\rm m}] \\ w & {\rm streamwise\ flow\ velocity\ [m\ s^{-1}]} \\ \end{array}$

x, y, z coordinates [μ m].

 μ viscosity [N s⁻¹ m⁻²].

 ρ density [kg m⁻³]

Greek letters

Subscripts

- 1 lower layer channel
- 2 upper layer channel.

1. Introduction

The heat removal problem has become an important factor in the advancement of microelectronics due to both drastically increased integration density of chips in digital devices as well as an increased current-voltage handling capability of power electronic devices. The task of removing a large amount of dispersed heat from a constrained, small space is often beyond the capability of conventional cooling techniques, therefore, new methods with heat removal capabilities at least one order larger than that of conventional ones are required.

One of the new techniques is based on the microchannel heat sink design. Since the early works of Tuckerman et al. [1], micro-channel heat sink has been studied and tested as a high performance and compact cooling scheme in microelectronics cooling applications. It is shown that the thermal resistance as low as 0.03°C/W is obtainable for micro-channel heat sinks, which is substantially lower than the conventional channel-sized heat sinks. Among relevant literature there are some exper-

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imental works as those given by Tuckerman et al. [1], Missaggia et al. [2] and Kleiner et al. [3] and some numerical works given by Samalam [4] and Weisberg et al. [5]. Design factors that have been studied include coolant selection (air [3] and liquid coolant [5]), inclusion of phase change (one phase and two phase by Bowers and Mudawar [6]), and structural optimization by Knight et al. [7].

One drawback of micro-channel heat sink is the relatively higher temperature rise along the micro-channels compared to that for the traditional heat sink designs. In the micro-channel heat sink, the large amount of heat generated by the semiconductor chips is carried out from the package by a relatively small amount of coolant so the coolant exists at a relatively high temperature.

This undesirable temperature gradient is an important consideration in the design of an electronic cooling scheme. A large temperature rise produces thermal stresses in chips and packages due to the coefficient of thermal expansion (CTE) mismatch among different materials thus undermining device reliability. Furthermore, a large temperature gradient is undesirable for the electrical performance since many electrical parameters are adversely affected by a substantial temperature rise. For instance in power electronic devices, electrical-thermal instability and thermal breakdown could occur within a high temperature region.

In the one-layered micro-channel heat sink design, bulk temperature rise along the channels can be controlled by increasing the pressure drop across the channels. A larger pressure drop forces coolant to move faster through the channel, thereby, requiring more powerful pumping power supply, generating more noise and requiring bulkier packaging. Two-phase micro-channel heat sink is an alternative method for eliminating the temperature variations, in which the utilization of latent heat can achieve a uniform temperature profile on the heating surface [6]. However, a two-phase scheme has several drawbacks such as a complicated structure and a much larger pressure drop required for the gas—liquid mixture to flow inside the minute conduits.

The objective of this work is to reduce the undesired temperature variation in the streamwise direction for the micro-channel heat sink by a conceptual design improvement, instead of increasing the pressure drop. The design concept proposed in this work is based on stacking two layers of micro-channel heat sink structures, one atop the other, with coolant flow in the opposite direction in each of the micro-channel layers, as illustrated in Fig. 1(a). For such an arrangement, streamwise temperature rise for the coolant and the substrate in each layer are compensated through conduction between the two layers, resulting in a substantially reduced temperature gradient. The tooling set-up for such a proposed two-layered structure is shown in Fig. 1(b). The flow loop in Fig. 1(b) is similar to the one designed for the one-layered micro-

channel heat sink, except that the flow loop bifurcates at the two ends of the heat sink test module and allows the coolant to flow from opposite directions into each of the channels.

In this work, the thermal performance of the proposed two-layered micro-channel heat sink is examined numerically using a finite element method and optimization issues for design parameters are addressed as well. Although a one-layered micro-channel heat sink has been extensively studied, to the best of the author's knowledge, the proposed two-layered structure concept has never been reported.

2. Computational model

Figure 2 illustrates the three-dimensional computational domain that is simplified based on the symmetry considerations. As shown in Fig. 2, the heat sink has two layers of rectangular channels and fins. The width and height of each of the channels are $2W_{\rm ch}$ and $H_{\rm ch}$ respectively. The width and height of each of the vertical fins are $2W_{\rm fin}$ and $H_{\rm ch}$ and the width and height of each of the horizontal fins are $2(W_{\rm fin} + W_{\rm ch})$ and $H_{\rm ba}$, respectively. The length of the channel in the streamwise direction is L. Flow in the bottom channel is along the z direction and flow at the top channel is opposite to the z direction.

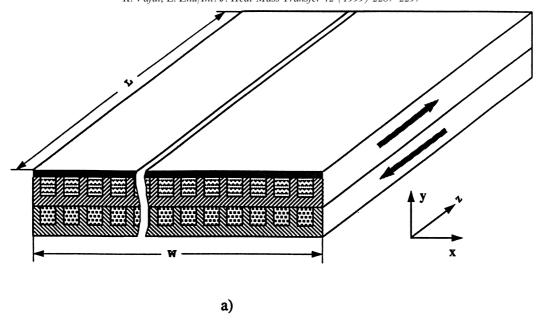
Previous numerical works on the one-layered microchannel heat sink are mostly based on a two-dimensional model. However, a three-dimensional model is necessary in this work because the temperature distribution in the two-layered structure is expected to be three-dimensional, along with a non-linear distribution in a streamwise direction. For the fin region shown in Fig. 2, the three-dimensional conduction equation with constant thermal conductivity as given below is utilized.

$$\nabla^2 T = 0. (1)$$

The flow in each of the micro-channels is assumed to be uni-directional. That is, the inlet and outlet hydraulic effects are neglected. This assumption can be justified by noting that the inlet and the outlet dimensions of the micro-channel's cross section are at least an order of magnitude less than the length of the channel. This allows the attainment of an analytical expression for the velocity profile. The temperature distribution within the micro channel can then be obtained from the following energy equation

$$w\rho c_{\rm p} \frac{\partial T}{\partial z} = k\nabla^2 T + \mu \left[\left(\frac{\partial w}{\partial x} \right)^2 + \left(\frac{\partial w}{\partial y} \right)^2 \right]$$
 (2)

where w is the streamwise velocity component. As mentioned before, the velocity distribution inside the channel, w, can be obtained analytically as [5]



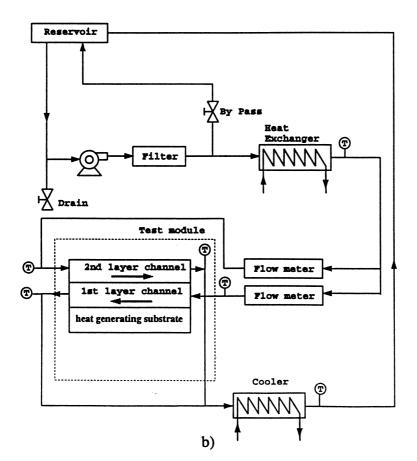


Fig. 1. (a) Schematic for the two-layered micro-channel heat sink concept. (b) The cooling set-up of the proposed two-layered structure.

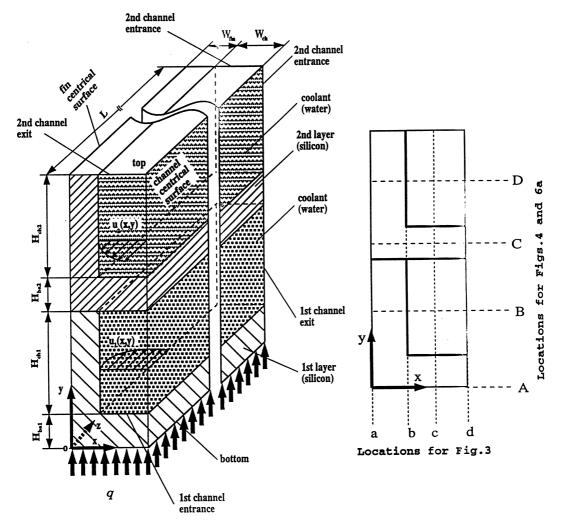


Fig. 2. Computational domain based on symmetry considerations.

$$w(x,y) = \frac{48}{\pi^3} \sum_{i=1}^{\infty} (-1)^{i-1} \left\{ 1 - \frac{\cosh\left[\pi(2i-1)x/H_{\text{ch}1,2}\right]}{\cosh\left[\pi(2i-1)W_{\text{ch}}/H_{\text{ch}1,2}\right]} \right\}$$

$$\times \left\{ \cosh\left[\pi(2i-1)W_{\text{ch}}/H_{\text{ch}1,2}\right]/(2i-1)^2 \right\}$$

$$\left\{ 1 - \frac{96}{\pi^5} \frac{H_{\text{ch}1,2}}{W_{\text{ch}}} \sum_{j=1}^{\infty} \frac{\tanh\left[\pi(2i-1)W_{\text{ch}}/H_{\text{ch}1,2}\right]}{(2i-1)^5} \right\}$$
(3)

where the average velocity is given by

$$\bar{w} = -\frac{H_{\text{ch1,2}}^{2}}{12\pi} \left(\frac{\mathrm{d}p}{\mathrm{d}z} \right) \left\{ 1 - \frac{96}{\pi^{5}} \frac{H_{\text{ch1,2}}}{W_{\text{ch}}} \sum_{j=1}^{\infty} \times \frac{\tanh\left[\pi(2i-1)W_{\text{ch}}/H_{\text{ch1,2}}\right]}{(2i-1)^{5}} \right\}$$
(4)

and where $W_{\rm ch}$ is the channel width and $H_{\rm ch1,2}$ is the channel height for either the lower or upper layer.

Based on the symmetry conditions, the central surface of the fin and the channel are taken as adiabatic. The top surface of the system is assumed to be adiabatic as well, because it is usually covered by thermally insulated materials. A uniform heat flux, q, is applied to the bottom of the composite micro-channel which is attached to the heat-generating chip. The coolant is assumed to enter each of the micro-channels at a constant temperature. Adiabatic conditions are utilized at the exits of each of the micro-channels. This is because conductive heat transfer at the exit is estimated to be less than 1% of the imposed heat transfer at the bottom surface. The micro-channel heat sink is assumed to be made of silicon and water is used as the coolant.

The Reynolds number (Re) is defined in the usual manner as

$$Re = \frac{\rho \bar{w} D_{1,2}}{u} \tag{5}$$

where $D_{1,2}$ are the hydraulic diameters of the lower and the upper channels, respectively, which are defined by

$$D_{1,2} = \frac{4W_{\rm ch}H_{\rm ch1,2}}{W_{\rm ch}+H_{\rm ch1,2}}.$$
 (6)

The numerical scheme is based on the finite element discretization using Galerkin method of weighted residuals. Eight-node iso-parametric brick elements are used in the computations. The application of the Galerkin based FEM is well described by Taylor and Hood [8], and its application in the finite element program used in the present work is documented [9]. A normal case ran

for about four hours on a R-10000 silicon graphic workstation.

3. Results and discussions

The temperature profiles in the y direction at various x and z locations are plotted in Fig. 3. The parameters used in generating Figs. 3–6 which are based on some typical generic conditions for these type of applications are listed in Table 1. Figures 3(a), 3(b) and 3(c) correspond to z=0, 0.5L, L which in turn signify the lower layer inlet/upper layer outlet, middle of the channel, and lower layer outlet/upper layer inlet, respectively. The curves in each figure correspond to different x locations labeled as (a), (b), (c) and (d) in Fig. 3.

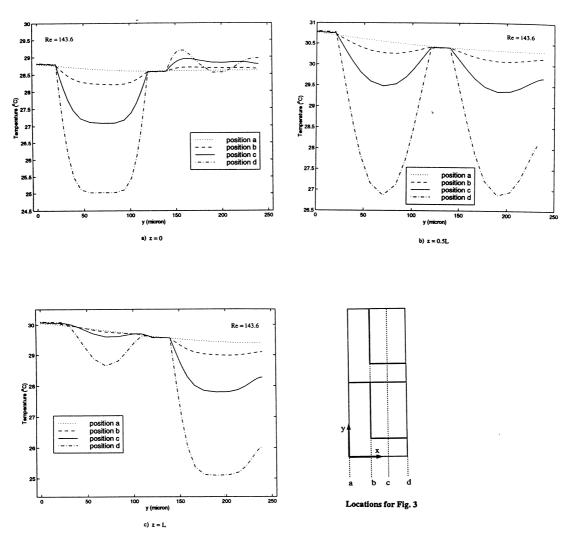


Fig. 3. Temperature distribution in normal (y) direction at different x and z locations.

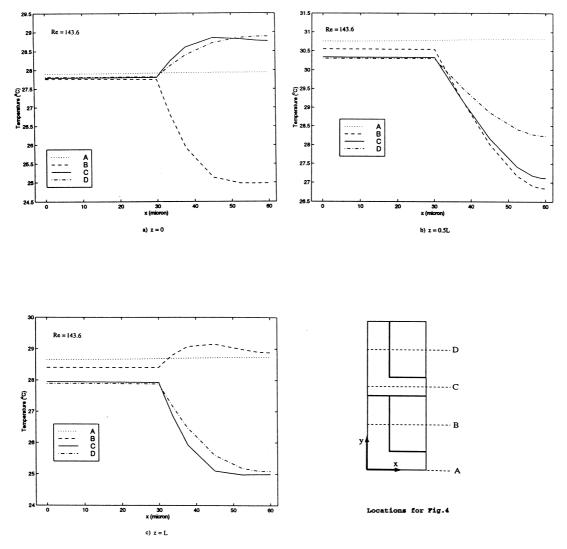
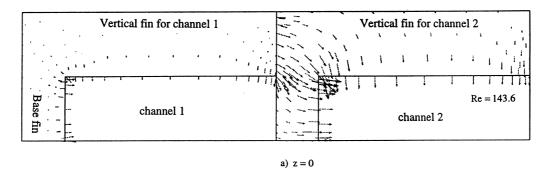


Fig. 4. Temperature distribution in *x* direction.

The temperature in the base section is virtually constant because of the large ratio of thermal conductivity of silicon to that of water, implying that the conduction contribution to the overall thermal resistance in the *y* direction is insignificant and the thickness of the silicon at the base is not a crucial factor for the thermal performance of the heat sink. The fluid temperature in the central channel domain is basically constant until it reaches the boundary layer region at which point the fluid temperature rises rapidly towards the solid surface temperature.

Due to the conduction effect on the coolant, the upper and lower layers of the fluid have apparent temperature difference. At the two ends of the channels, there are regions where the outlet coolant temperature for one layer is higher than the temperature of the surrounding substrate cooled by the other coolant layer at its inlet. This implies that for each layer, heat dissipates from substrate to coolant at its inlet, as it does in one-layered micro-channel heat sink, but in addition heat transfer also occurs from the heated coolant to the substrate around its outlet. This feature is unique and occurs only in the two-layered heat sink design. It is also observed that temperature distribution in the y direction at various z positions are quite different. For example, the temperature in solid domain remains almost constant with height in Figs. 3(b) and 3(c), but in Fig. 3(a), it drops only until the mid-section of the lower channel and then it increases gradually with height. This is also different from a one-layered structure where the temperature profile should be invariant in streamwise distance.

Temperature profiles in the x direction are plotted in Fig. 4. Figures 4(a), 4(b) and 4(c) correspond to different



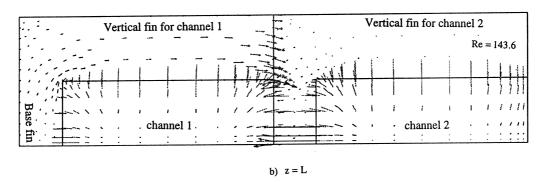


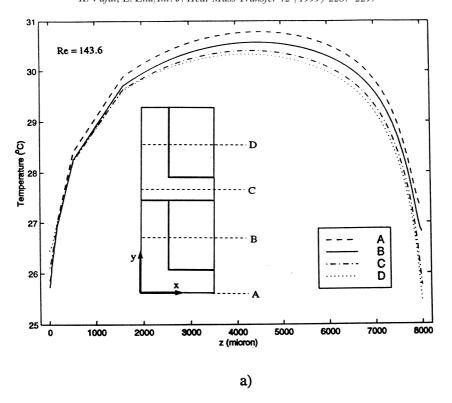
Fig. 5. Heat flux vector plots at z = 0 and z = L for Re = 143.6.

z positions of 0, 0.5L, and L respectively. The curves in each figure correspond to different y locations labeled as (A), (B), (C) and (D) in Fig. 4. All curves in Fig. 4 change abruptly at the interface between the fin and the channel. The temperature profiles at the base of the lower layer, as represented by the curves in Figs. 4(a) and 4(b), display a temperature increase from the central surface of the fin to the central surface of the channel. This is indicative of a heat flux parallel to the base surface, the direction of which is from the channel side to the fin side.

The heat flux paths suggested in Fig. 4 can be seen clearly in the heat flux vector plots displayed in Fig. 5. Figure 5 illustrates that the heat input at the bottom surface of the heat sink is dissipated in two ways: a "direct" path which is by conduction through the base layer and convection from the lower layer of the channel; and an "indirect" path which is by conduction through the vertical fin and the upper base layer as well as convection from the upper layer channel. As seen in Fig. 5, the "direct" path dominates the heat transfer process around z = 0 whereas around z = L the main heat flux flows through the "indirect" path. This is because around z = 0, as seen in Fig. 5(a), which corresponds to the lower channel inlet and the upper channel outlet, the coolant in the lower channel is cooler than that in the upper channel, so the "direct" heat flux path is more significant than the "indirect" path. The opposite scenario is the case around z = L as seen in Fig. 5(b) where the heat flux along the "indirect" path is dominant.

Figure 6(a) shows the temperature distribution along the channels, corresponding to the central surface of the fin. The four curves in Fig. 6(a) correspond to different y locations labeled as (A), (B), (C) and (D). The results for the streamwise temperature distribution within the one-layered channel structure is shown in Fig. 6(b). For comparative purposes, the pressure drop and other parameters are kept the same as those used in the two-layered structure.

As shown in Fig. 6(a), the streamwise temperature distribution at different y positions, denoted by the curves (A)–(D), are close to each other, with maximum value occurring approximately at the center of the channels regardless of the y location. The maximum temperature difference in the streamwise direction in the two-layered structure is around 5°C as seen in Fig. 6(a). The benefits of the two layered micro-channel system become apparent when the maximum temperature difference for the two-layered system shown in Fig. 6(a) is compared with that for the one-layered channel shown in Fig. 6(b). As seen in Fig. 6(b), the maximum temperature difference for the one-layered micro-channel system is about 15°C or about 300% higher than that for the two-layered micro-channel system for the same set of parameters. Therefore, the two-layered micro-channel heat sink can



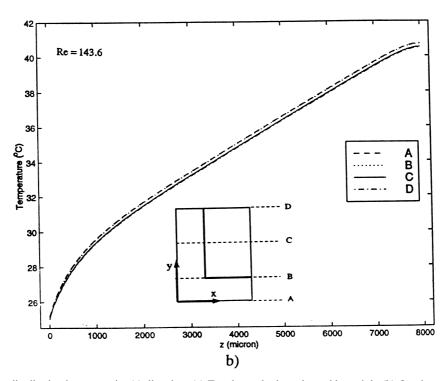


Fig. 6. Temperature distribution in streamwise (z) direction. (a) Two-layered micro-channel heat sink. (b) One-layered micro-channel heat sink.

Table 1 Computational parameters used in generating Figs. 3–6

$W_{\rm ch} (\mu \rm m)$	$W_{\rm fin}$ (μ m)	$H_{\rm ch1}~(\mu{\rm m})$	$H_{\rm ch2}$ (μ m)
30	30	100	100
$D_{\rm h1}~(\mu{ m m})$	$D_{\mathrm{h2}}~(\mu\mathrm{m})$	$L(\mu m)$	ΔP (Pa)
92.3	92.3	800	0.24
Re	$\bar{w} \ (m \ s^{-1})$	$q (\mathrm{W} \mathrm{cm}^{-2})$	T_{inlet} (°C)
143.6	1.38	30	25

reduce the streamwise temperature difference more effectively than the traditional one-layered design.

4. Optimization

In this section, the optimization of the double-layered micro-channel heat sink design is studied. The parameters that are used to evaluate the thermal performance of a heat sink are the normal and the streamwise thermal resistances. The normal thermal resistance is defined as

$$R_{\rm th}^{\rm n} = \frac{\Delta T_{\rm max}^{\rm n}}{q} \tag{7}$$

where $\Delta T_{\rm max}^{\rm n}$ is the maximum temperature rise from the coolant in the bottom channel to the bottom surface of the heat sink that is attached to the heat-generating chips and q is the heat flux at the bottom surface. The streamwise thermal resistance is related to the rate of the streamwise temperature gradient and the applied heat flux, and is defined by

$$R_{\rm th}^{\rm z} = \frac{\Delta T_{\rm max}^{\rm z}}{q} \tag{8}$$

where ΔT_{max}^z denotes the maximum temperature increase along the z direction on the bottom surface of the silicon.

The task of optimization is to find the optimal design parameters that would minimize the cited two thermal resistances in the normal and streamwise directions. Pertinent geometrical parameters related to the thermal performance of a two-layered micro-channel heat sink are the ratio of channel height to channel width, the ratio of fin width to the channel width, the size of the channels, and the streamwise channel length of the heat sink. It has been established by Weisberg and his co-workers [5] that in the one-layered structure, smaller channel size and larger ratio of channel height to width can reduce the normal thermal resistance, as long as such a design will satisfy the structural consideration. These observations are also valid for two-layered structures. Weisberg et al. [5] reported that the optimal ratio of channel width to fin width is about 1.0 for the one-layered structure. Their result, however, may not be true for a two-layered design because the heat dissipation pattern through the channel and the fin, as discussed previously, are quite different between the one-layered and two-layered structures.

The effects of the $W_{\rm ch}/W_{\rm fin}$ on the normal thermal resistance, $R_{\rm th}^{\rm n}$, is shown in Fig. 7 for both the two-layered and the one-layered structures under the same operating conditions. It should be noted that $W_{\rm ch}/W_{\rm fin}$ variations did not have a significant impact on the streamwise thermal resistance, R_{th}^z . In the two-layered structure, the minimum $R_{\rm th}^{\rm n}$ is obtained at the $W_{\rm ch}/W_{\rm fin}$ ratio of 0.6, compared to 1.0 for the one-layered structure. This means that the fins for an optimal two-layered micro channel heat sink will be thicker than the fins of the one-layered structure having the same patch width. The reason for this difference in fin thickness is due to the fact that the fin for the bottom layer in the two-layered structure transports a higher percentage of heat (to both the bottom and top channels) than the fin for the one-layered structure; and a thicker fin apparently facilitates this heat transfer. It should be noted that the normal thermal resistance varies only slightly in the range of $W_{\rm ch}/W_{\rm fin}$ less than 0.6. Considering the difficulty of fabrication of smaller channels, $W_{\rm ch}/W_{\rm fin}$ of 0.6 is therefore recommended as the optimal ratio for the practical design of a two-layered micro channel heat sink.

The effects of the ratio of the pressure drops in the bottom and top layer, $\Delta P_1/\Delta P_2$ on the streamwise thermal resistance, R_{th}^{z} , for both the one-layered and two-layered structures are also plotted in Fig. 7. It should be noted that $\Delta P_1/\Delta P_2$ variations did not have a significant impact on the normal thermal resistance, R_{th}^{n} . In order to compare the two cases, once again the same heat flux is used for both cases and the flow rate used for the one-layered case is set equal to the sum of the flow rates in the top and the bottom layers for the two-layered case. It should be noted that for the one-layered channels, the amount of heat carried out by the coolant flow is proportional to the flow rate, which in turn, is proportional to the pressure drop applied across the channels. For the twolayered channels, given the total amount of the flow rate of coolant in both channels, it requires a knowledge of the optimal pressure drop ratio that can minimize the streamwise thermal resistance. It can be observed in Fig. 7 that as $\Delta p_1/\Delta p_2$ increases from 0.7 to about 3.0, the streamwise thermal resistance rate decreases significantly from 17.2°C/cm W to 11.4°C/cm W, however, further increase of this pressure ratio does not reduce the thermal resistance much further. Based on this fact, a pressure ratio of 2.5-3.0 is recommended as an optimal ratio for a practical two-layered design. This optimal pressure ratio is also found to hold true for the partially heated cases as shown in Fig. 7.

Another geometric parameter that can be optimized in the design of a two-layered micro channel heat sink is the channel length. The streamwise temperature distribution in Fig. 6 indicates that a large portion of the temperature rise occurs at the two ends of the channels due to rela-

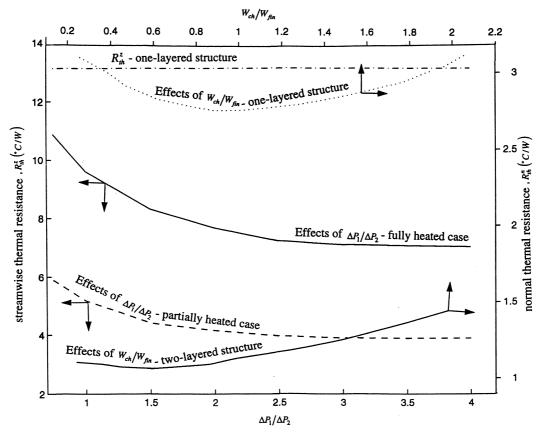


Fig. 7. Effects of $W_{\rm ch}/W_{\rm fin}$ and $\Delta P_1/\Delta P_2$ on the normal and streamwise thermal resistances.

tively intense heat transfer between the low-temperature coolant of one layer at its inlet and the high-temperature coolant of the opposite layer at its outlet within a short distance. This implies that the two-layered micro channel can be constructed with a longer length in the streamwise direction, so that the heat-generating semiconductor chips can be mounted in the mid section of the heat sink to avoid the two high temperature gradient regions at both ends of the channels. This idea is implemented in our calculations by modifying the boundary condition at the bottom of the heat sink to one of partial heating, specifically, three-fifths of the total length at the mid section of the channel is exposed to heat flux. Figure 8 shows the streamwise temperature distribution at x = 40and y = 0 (at the bottom surface) of this partially heated case. The overall channel length is kept at 8000 μ m, which is the same as the length used for the fully heated case, however, the heated region is restricted from z = 1600 μm to $z = 6400 \mu m$. All other parameters are kept the same as for the fully heated case discussed previously. The temperature increases from 27.6°C to 29.5°C and decreases from 29.7°C to 25.0°C within the two end regions. In the heated region where the semiconductor chip is mounted, the temperature variation is merely about 0.5° C. This demonstrates that a longer micro channel design can indeed substantially reduce the streamwise temperature variation. It should be pointed out that the one-layered design does not have these characteristics. A longer channel length in the one-layered structure will not affect the streamwise temperature rise due to the absence of the heat transfer mechanism between the coolant flows within the top and the bottom layers.

5. Conclusions

The design of two-layered micro-channel heat sinks with opposite flow direction on each layer is proposed in this work. The thermal performance of the proposed structures are modelled numerically. In particular, this study is focused on the temperature distribution, the thermal resistance, and the optimization of geometrical design parameters. Although the power supply system of the two-layered design will not be significantly more complicated than the traditional one-layered design, it is found that the streamwise temperature rise on the base

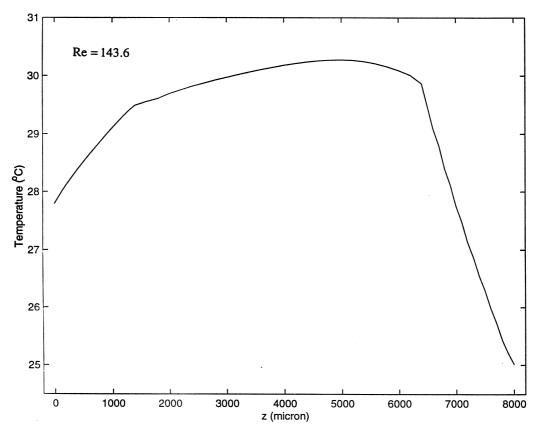


Fig. 8. Streamwise temperature distribution for partially heated cases.

surface is significantly reduced compared to the one-layered structure, and therefore the pressure drop required for the two-layered design can be much smaller than that of the one-layered design when small temperature variation is required for the semiconductor chips. The present work establishes the process for a tailored design and optimization of a two-layered micro channel heat sink. Our result demonstrates that the two-layered micro channel heat sink design is a substantial improvement over a conventional one-layered micro channel heat sink.

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